Name ______________________________

CALCULUS AB

3.3 – Increasing and Decreasing Functions and the First Derivative Test

Definition:

1) A function f is **increasing** on an interval if for any two numbers

x_1 and x_2 in the interval, $x_1 < x_2$ implies _____________.

2) A function f is **decreasing** on an interval if for any two numbers

x_1 and x_2 in the interval, $x_1 < x_2$ implies _____________.

Test for Increasing and Decreasing Functions

1) If $f'(x) > 0$ for all x in (a, b), then ____________________________.

2) If $f'(x) < 0$ for all x in (a, b), then ____________________________.

3) If $f'(x) = 0$ for all x in (a, b), then ____________________________.

First Derivative Test:

Let c be a **critical number** of a function f that is continuous on an open interval I containing c.
If f is differentiable on the interval, except possibly at c, then $(c, f(c))$ can be classified as follows.

1) If $f'(x)$ changes from _______________ at $x = \text{c}$, then $(c, f(c))$ is a __________________ of *f*.

2) If $f'(x)$ changes from _______________ at $x = c$, then $(c, f(c))$ is a __________________ of *f*.

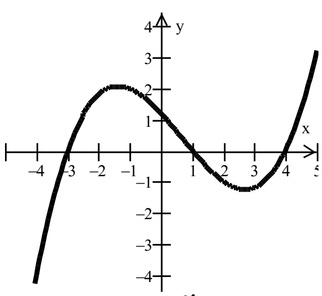
Ex. Find the intervals where *f* is increasing and decreasing, identify all points that are relative maximum and minimum points, and justify your answers. Use your results to sketch the graph.

(a) $f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$

(b) $f(x)=\left(x^2-4\right)^{2/3}$

(c) $f(x)=\dfrac{x^2}{2x-1}$

<u>Ex</u>. Use the graph of f' to:
(a) identify the interval(s) on which the graph of f is increasing and decreasing;
(b) estimate the value(s) of x at which the graph of f has a relative maximum or minimum.
Justify your answers.



Graph of f'

Let g be the function defined by $g(x)=x^4+4x^3$. How many relative extrema does g have?

Ex. The function $s(t)=t^2-7t+10$ describes the motion of a particle along a line.

(a) Find the velocity function of the particle at any time $t \geq 0$.

(b) Identify the time interval(s) in which the particle is moving in a positive direction. Justify.

(c) Identify the time interval(s) in which the particle is moving in a negative direction. Justify.

(d) Identify the time(s) at which the particle changes direction. Justify.

AP Style Practice

1. The function $y=g(x)$ is differentiable and decreasing for all real numbers. On what intervals is the function $y=g(x^3-6x^2)$ increasing?

2. Let f be the function given by $f(x)=3-2x$. If g is a function with derivative given by $g'(x)=f(x)f'(x)(x-3)$, on what intervals is g increasing?

3. What are all values of x for which the function f defined by $f(x)=(x^2-3)e^{-x}$ is increasing?

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

4. The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on what intervals?

CALCULUS AB

Section 3.4 – Concavity and the 2nd Derivative Test

Definition of Concavity:
Let f be differentiable on an open interval I.

1) The graph of f is **concave upward** on an interval I

if f' is __________________ on the interval.

2) The graph of f is **concave downward** on an interval I

if f' is __________________ on the interval.

Test for Concavity:
Let f be a function whose second derivative exists on an open interval I.

1) If $f''(x) > 0$ for all x in I, then the graph of f is _________________________ in I.

2) If $f''(x) < 0$ for all x in I, then the graph of f is _________________________ in I.

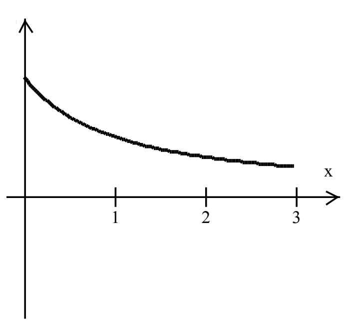
Definition of an Inflection Point:
A function f has an inflection point at $(c, f(c))$

1) if ________________ **OR** if ________________ **AND**

2) if f'' changes sign from ____________________ or ____________________ at $x = c$

OR if $f'(x)$ changes from ____________________ or ____________________ at $x = c$.

Ex. The graph of f is shown below. State the signs of f' and f'' on the interval (0, 3).



Ex. Given the function $f(x) = x^4 - 4x^3$, find:
a) the intervals where f is increasing and decreasing
b) the relative extrema
c) the intervals where f is concave up and concave down
d) the inflection points
Justify your answers, and use the information you found to sketch the graph of f.

Ex. Sketch the graph of a continuous function f with the given characteristics:

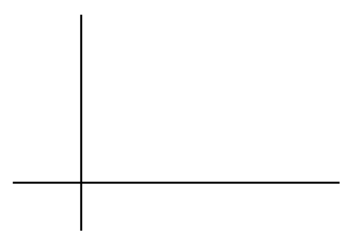
(a) $f(0) = f(2) = 0$

$f'(x) > 0$ if $x < 1$

$f'(1) = 0$

$f'(x) < 0$ if $x > 1$

$f''(x) < 0$



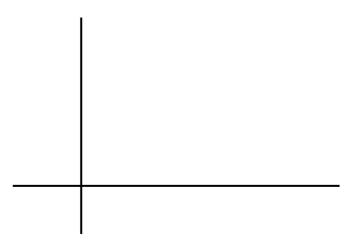
(b) $f(0) = f(2) = 0$

$f'(x) > 0$ if $x < 1$

$f'(1)$ is undefined

$f'(x) < 0$ if $x > 1$

$f''(x) > 0$



Second Derivative Test:
Let f be a function such that the second derivative of f exists on an open interval containing c.

1) If $f'(c)=0$ and $f''(c)>0$, then $f(c)$ is a ___________________ of f.

2) If $f'(c)=0$ and $f''(c)<0$, then $f(c)$ is a ___________________ of f.

Ex. Use the Second Derivative Test, if possible, to find the relative extrema of $f(x)=-3x^5+5x^3$. Justify your answer.

Ex. Suppose that the function f has a continuous second derivative for all x, and that $f(3)=-4,\ f'(3)=1,\ f''(3)=-2$. Let g be a function whose derivative is given by $g'(x)=\left(x^2-9\right)\left(2f(x)+5f'(x)\right)$ for all x. Does g have a local maximum or a local minimum at $x=3$? Justify your answer.